MBS-01-50-4

MBS-01 Seat No. ________

M. Phil. (Sem. II) (CBCS) Examination

April / May – 2018

Mathematics : CMT - 20001

(Topology)

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1) There are **five** questions in this paper.

(2) All questions are **compulsory**.

1 Fill in the blanks: (Each question carries **two** marks) **14**

(a) If $f : X \to \mathbb{R}$ is a continuous function then f^{1} ({1}) is a ______ set.

(b) If I is a Z - ideal which contains a prime ideal then I is a ______ ideal.

(c) Every ______ maximal ideal in $C^*(\mathbb{N})$ contains the function $j(n)$.

(d) In a normal space X every ______ subset is C-embedded in X.

(e) In $C(\mathbb{N})$ every ideal is a ______ ideal.

(f) If A and B are completely separated in X then A and B are contained in disjoint ______ sets.

(g) The space of natural numbers is ______ embedded in its Stone - Cech compactification.

2 Attempt any **three** of the following : **24**

a) Prove that an ideal M is a maximal ideal if and only if Z (M) is a Z - ultra filter.

b) State and prove the necessary and sufficient condition under which a subspace S of X is C^* - embedded in X.

c) i) Give an example of an ideal in $C(X)$ which is a Z - ideal.

ii) Prove for any ideal I, Z^{-1} (Z (I)) is a Z - ideal.

d) Prove that countable intersection of a zero sets is a zero set.

3 All are compulsory : **24**

a) Let $I = \{f \in C(\mathbb{R}) : Z(f) \textit{ is a neibhbourhood of } 0\}$. Show that I is a Z-ideal and it is not the intersection of maximal ideals containing it. **8**

b) Give an example of an ideal in $C^*(\mathbb{N})$ which is not the intersection of any ideal of $C(\mathbb{N})$ with $C^*(\mathbb{N})$. **6**

c) Suppose I is a Z-ideal which contains a prime ideal. Prove that I is a prime ideal in $C(X)$. **5**

d) Prove that in a compact space every z-filter has a cluster point. **5**

OR

3 All are compulsory : **24**

a) Let X be a compact Hausdorff space. **8**

i) Prove that the closure of an ideal I in $C(X)$ is an ideal in $C(X)$.

ii) Prove that every maximal ideal in $C(X)$ is closed. [$C(X)$= the Banach algebra of all complex valued continuous functions on X].

b) Prove that two sets are completely separated in X if and only if they are contained in disjoint zero sets of X. **5**

c) Prove that every prime ideal of $C(X)$ is contained in a unique maximal ideal of $C(X)$. **5**

d) Prove that $\beta(X)$ is disconnected if and only if X is disconnected. **6**

4 Attempt any **three** of the following : **24**

a) Prove that a space X is compact if and only if every maximal ideal in $C^*(X)$ is fixed.

b) Suppose X is a dense subspace of T and X is C^* - embedded in T. Prove that

i) If Z_1 and Z_2 are disjoint zero sets in X then $Cl_T(Z_1)$ and $Cl_T(Z_2)$ are disjoint.

ii) If Z_1 and Z_2 are zero sets in X then $Cl_T(Z_1 \cap Z_2) = Cl_T(Z_1) \cap Cl_T(Z_2)$.

c) Let $C(X)$ be the banach algebra of all complex valued continuous functions defined on a compact hausdorff space X. Prove that there is a one-one correspondence between the non-empty closed subsets of X and closed ideals of $C(X)$.

d) Prove that a subset S of $\mathbb{R}$ is C-embedded in $\mathbb{R}$ if and only if it is a zero set of $\mathbb{R}$.

5 Do as directed : (Each question carries **two** marks) **14**

a) Give reasons why $\mathbb{R} - \{0\}$ is not a zero set.

b) Give a continuous function $f : \mathbb{R} \to \mathbb{R}$ for which $Z(f)$ is a countable infinite set.

c) Suppose $f \in C(X)$ and $A = \{x \in X \mid f(x) > \frac{1}{2}\}$. Is A zero set? Give reason.

d) Suppose $f(x) = x$ for all x in $\mathbb{R}$. Let I be the principal ideal generated by $f(x)$. Give a function g in $C(X)$ such that $g \in Z^{-1}(Z(I))$ but $g \notin I$.

e) Give the definition of a compactification of a space X and give the characteristic property of the Stone - Cech compactification of X.

f) Give an example of a free maximal ideal in $C(\mathbb{N})$.

g) Give a continuous function $f : \mathbb{N} \to \mathbb{R}$ such that f cannot be extended to a continuous function $g : \beta(\mathbb{N}) \to \mathbb{R}$.